A POINT ELECTRICAL DISCHARGE IN A LIQUID

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The release of energy in a discharge channel causes the formation of a plasma at high temperature and pressure, which leads to the rapid expansion of the channel, accompanied by the emission of a compression wave and the subsequent formation of a pulsating gas bubble. Depending on the parameters of the discharge circuit the discharges produced can have hydrodynamic characteristics corresponding to the long-cylinder model [1], the short-cylinder model [2], or the spherical model discussed in this paper. This model applies in the case in which the length l of the discharge gap is small in comparision with the characteristic radius R_0 of the channel, which, in turn, is small in comparison with the characteristic length λ of the emitted wave. The hydrodynamic and electrical characteristics of a discharge are connected by the energy-balance equation [3], whose solution enables the determination of the law of channel expansion, the pressure in it, the parameters of the compression wave, and other hydrodynamic quantities.

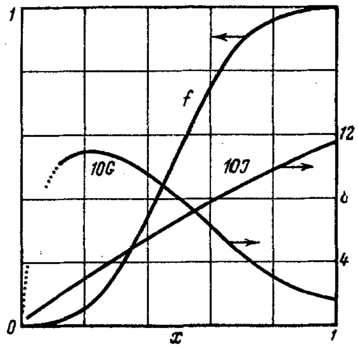

Fig. 1

1. Restricting ourselves to the consideration of electrical discharges that are not too strong, we will follow [3] and assume that the energy E released in the channel is expended on increasing the internal energy W of the plasma and on performing work A on the surroundings. The balance equation in this approximation has the form

$$W + A = E, \qquad W = p_C V / (\gamma - 1). \tag{1.1}$$

Here p_C is the pressure in the channel with volume V, γ is the effective adiabatic exponent of the plasma [4], which for water is equal to 1.2 [3],

$$A = \int_{V_0}^{V} p_C\, dV, \qquad E = \int_0^t IU\, dt. \tag{1.2}$$

Here I and U are the current and voltage on the interelectrode gap.

Regarding the surface of the channel as impermeable [5] and neglecting the magnetic pressure we can find the pressure on the surface of the channel as a function of its radius R from the solution of the hydrodynamic problem of expansion of a sphere in a liquid. The approximate solution of this problem, valid for low (in comparison with that of sound) velocities of expansion of the channel, when the liquid can be regarded as incompressible, is written in the form [6]

$$p_C - p_0 = \rho V^{\cdot\cdot}/4\pi R - \rho V^{\cdot 2}/(4\pi)^2 R^4. \tag{1.3}$$

Neglecting $p_0 \ll p_C$, we have for the work of expansion from (1.2)

$$A = \rho V^{\cdot 2}/8\pi R. \tag{1.4}$$

Now, substituting (1.3) and (1.4) into (1.1), we obtain

$$VV^{\cdot\cdot} + {}^1/_2 (\gamma - {}^4/_3) V^{\cdot 2} = (\gamma - 1)\, \rho^{-1} 4\pi R E. \tag{1.5}$$

We introduce the dimensionless variables $x = t/T$, $y = R/R_0$, and the function $f(x) = E(x)/E(T)$. Here T is a characteristic time scale, equal, for instance, to the duration of the discharge; R_0 is an arbitrary (for the present) unit of length.

If we neglect the small second term in Eq. (1.5) and select R_0 on the basis of the condition that the coefficient on the right side of the obtained dimensionless equation becomes unity, we can write Eq. (1.5) in the form

$$y^2 dy^{\cdot} y^2 / dx = f(x) \tag{1.6}$$

and in this case R_0 is given by the expression

$$R_0^5 = {}^3/_4 T^2 E(T) \;\; (\gamma - 1) / \rho\pi. \tag{1.7}$$

We choose the initial conditions in the form

$$x = 0, \qquad y = y_0, \qquad y^{\cdot} = 0, \tag{1.8}$$

where y_0 is the initial radius of the channel, which is chosen small in comparison with its value at the end of the discharge. A more accurate definition of y_0 is unnecessary, since the expansion process depends on the conditions of energy release in the channel and not on the initial conditions, which the system rapidly "forgets."

2. If we know, from experiment for instance, the function $f(x)$, which determines the energy release in the channel, then, by integrating (1.6), we can find the law of expansion of the channel

$$R = R_0 y \tag{2.1}$$

and the pressure in the channel

$$p_C = \rho \frac{R_0^2}{T^2}\left[\frac{f(x)}{y^3} - \frac{z^2}{2y^4}\right], \qquad z = y^{\cdot} y^2. \tag{2.2}$$

At low velocities of channel expansion the compressibility of the fluid has an insignificant effect on the law of its expansion

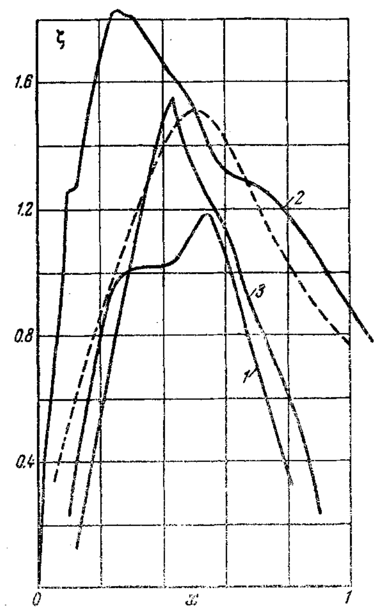

Fig. 2

and, hence, solution (2.1), found for an incompressible fluid, can be used to calculate the pressure in the compression wave

$$p = \rho \frac{V^{\cdot\cdot}}{4\pi r} = \rho \frac{R_0^2}{T^2} \frac{R_0}{r} \frac{f(x)}{y^2}, \qquad X = x - \frac{r}{cT}. \tag{2.3}$$

Experiment shows that the function $f(x)$ for all discharges in a near-critical regime is practically the same and has the form shown in Fig. 1 [2]. Using this graph, we can integrate Eq. (1.6). The results of the calculation for the functions

$$J = R/R_0,\ G = p_k T^2 r/\rho R_0^3$$

are shown in Fig. 1, and for function $\zeta = f(x)/y^2 = pT^2 r/\rho R_0^3$ are shown by the dashed curve in Fig. 2. It is clear that the pressure in the compression wave, according to Fig. 2 (dashed curve), exceeds the dimensional coefficient in formula (2.3) by a factor of 1.5. Knowing the pressure in the emitted compression wave, we find the acoustic energy removed by this wave

$$W_a = 4\pi r^2 \int\limits_0^\infty \frac{p^2}{\rho c}\, dt = \frac{\pi\rho R_0^6}{cT^3} I ,$$

$$I = \int\limits_0^\infty \left[\frac{f(x)}{y^2}\right]^2 dx. \qquad (2.4)$$

We note that in the calculation of W_a integration can be taken to x = 1 in view of the rapid decrease in the integrand and then for function $f(x)$, shown in Fig. 1, we will have I = 1.17.

An important characteristic of the discharge will be its electo-acoustic efficiency

$$\eta = \frac{W_a}{E(T)} = 3(\gamma - 1)\frac{R_0}{cT} I =$$

$$= 4\pi\rho\left[\frac{3}{4\pi}\frac{\gamma-1}{\rho}\right]^{1.2}\frac{E(T)^{0.2}}{cT^{0.6}} I . \qquad (2.5)$$

It is clear that η is proportional to the velocity of expansion of the channel R_0/T, depends significantly on the effective adiabatic exponent for the discharge plasma, depends weakly on the amount of released energy, and increases with reduction in the duration of the discharge.

For future reference we note that the plasma internal energy is

$$W = \rho\frac{R_0^5}{T^2}\frac{4\pi}{3(\gamma-1)}\left[f(x) - \frac{z^2}{2y}\right]. \qquad (2.6)$$

The work performed by the channel is

$$A = 2\pi\rho\frac{R_0^5}{T^2} y^3 y'^2 . \qquad (2.7)$$

3. After the discharge a pulsating gas bubble is formed. To determine its characteristics we use the known expression describing the pulsations of a gas-filled spherical cavity in a liquid [7],

$$R'^2 = \frac{2}{3}\frac{p_1}{\rho}\left\{\frac{1}{\gamma_1 - 1}\left[1 - \left(\frac{R_1}{R}\right)^{3\gamma_1 - 3}\right]\left(\frac{R_1}{R}\right)^3 - \right.$$

$$\left. - \frac{p_0}{p_1}\left(1 - \frac{R_1^3}{R^3}\right)\right\}. \qquad (3.1)$$

Here R_1 and p_1 are the initial radius of the cavity and the pressure in it, p_0° is the equilibrium pressure in the liquid, and γ_1 is the adiabatic exponent of the gas in the cavity.

The values of R_1 and p_1 are determined from the condition that at the instant when the discharge ends (i.e., when x = 1) the radius of the cavity, the velocity of its expansion, and the pressure in it are equal, respectively, to the radius R of the channel, the velocity R' of its expansion, and the pressure p_c in it:

$$R = R_0\, y(1), \qquad R' = (R_0 / T)\, B ,$$
$$p_c = \rho R_0^2 A_1 / T^2.$$

Here A_1 and B—the values of the dimensionless functions in the expressions for the velocity and pressure at the instant x = 1—are equal to 0.42 and 0.81. A simple calculation leads to the following results:

$$R_1 = R(1)(1 + A_1)^{-\varkappa},$$

$$p_1 = \rho\frac{R_0^2}{T^2} A_1\left(1 + \frac{3}{2}\frac{B^2}{A_1}(\gamma_1 - 1)\right)^{3\varkappa\gamma},$$

$$\varkappa = \frac{1}{3(\gamma_1 - 1)} , \qquad (3.2)$$

which enable us now to find from (3.1) the motion of the gas sphere formed by the discharge. In particular, the energy of the pulsating bubble is

$$W^\circ = E\frac{\gamma - 1}{\gamma_1 - 1}\left\{A_1 y^3(1)\left[1 + \frac{3}{2}\frac{B_2}{A_1}(\gamma_1 - 1)\right]\right\}. \qquad (3.3)$$

A numerical calculation shows that the expression in the braces is close to unity (about 1.58) and, hence, the fraction of the en-

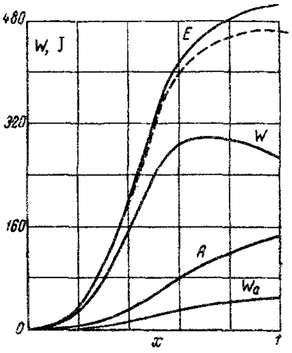

Fig. 3

ergy going towards pulsation of the bubble depends mainly on the difference in the values of the adiabatic exponents for the plasma and gas in the bubble.

Knowing W°, we can easily determine the period of pulsation of the bubble,

$$T^\circ = 1.135\, \rho^{1/2} p_0^{-5/6} W^{\circ 1/3} . \qquad (3.4)$$

4. In experimental conditions it is usually convenient to produce the discharge at the surface of a hard reflector, and not in free space. The effect of this surface can be taken into account in the calculation. We consider the case of a discharge occurring at a hard flat surface. If, as before, the interelectrode gap is small in comparison with R_0, and R_0 is small in comparison with the characteristic wavelength λ, the shape of the channel will be almost hemispherical. Insofar as effects associated with boundary-layer formation are concerned, such a discharge is equivalent to half of the spherical discharge in free space. Hence, it is clear that the hydrodynamic characteristics of the discharge near a plane will be the same as those of a free discharge with twice the energy. They can be calculated from formulas (2.1)–(2.7), if in the calculation of R_0 contained in these formulas E in expression (1.9) is replaced by 2E and, in addition, the values of the acoustic energy, the internal energy, and the work of the channel are halved, so that the energy in only one of the half-spaces is considered. To find the period of pulsation of the bubble formed in a discharge near a hard flat reflector we must replace W° in formula (3.4) by 2W°.

5. In view of the known universality of the normalized law of energy release $f(x)$, which is almost constant for all near-critical discharges, the only values required for calculation of the hydrodynamic characteristics of such discharges are those of the two parameters E and T, which, in turn, can be determined approximately by the relationships $T \approx 2\pi(LC)^{1/2}$ and $E(T) = (1/2)CU^2$. This provides the possibility of an approximate determination of the hydrodynamic characteristics from the parameters C, L, and U of the discharge circuit. These characteristics, as formulas (2.1)–(2.4) and (2.6), (2.7) show, depend in a universal manner on the dimensionless time x. Using the approximate expressions for T and E, we find for water

$$\eta \approx (\gamma - 1)^{1.2} U^{1.4} C^{-0.1} L^{-0.3} , \qquad (5.1)$$

i.e., the efficiency of the discharges is practically independent of the capacitance of the storage device and increases with increase in voltage or decrease in inductance. In addition, from (2.3) we find that the pressure in the compression wave increases in proportion to $C^{3/5}$ and $U^{6/5}$.

6. In the light of the above we consider some experimental results obtained in the investigation of discharges corresponding to the spherical model. For the production of such discharges the parameters of the discharge circuit can be chosen as follows. The length of the interelectrode gap is taken as a few millimeters, and the characteristic wavelength λ as a few centimeters. Then T will be a few tens of microseconds.

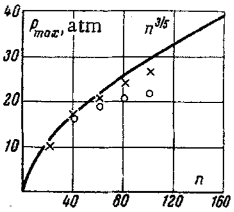

Fig. 4

Knowing the order of magnitude of T we evaluate the energy of the storage device from formula (1.7), taking $R_0 \approx 1$ cm $> l$, with the proviso that the discharge is produced near a hard reflector, in the center of which it is convenient to insert a dielectric plug with electrodes, the ends of which are flush with the reflector. Taking $T = 50 \cdot 10^{-6}$ sec and $\gamma = 1.2$, we find that the energy of the storage device will be approximately 500 J. The values of L, C, and the working voltage are still undetermined. If there is no strict requirement that L be reduced and it is taken as a few tens of microhenrys, then, assuming $T = 2\pi (LC)^{1/2}$ we find C equal to a few hundred microfarads and, hence, the voltage must be on the order of 1 kV. Discharge 1 and 2 correspond to a discharge circuit with such parameters.

	U kV	C μF	l cm	T μsec	E (τ) J	$^1/_2\, CU^2$ J	R_0 cm
1	1.6	320	0.1	33	376	412	0.82
2	1.2	816	0.3	50	510	588	1.04
3	15	1500	7.5	400	—	22600	6.6

At the boundary of applicability of the spherical model we find discharge 3, investigated in [8] for which the data are given in the third line.

For discharges 1 and 2 we recorded oscillograms of the discharge current and the voltage on the interelectrode gap and from them we found the functions $f(x)$, which were practically the same as $f(x)$ shown in Fig. 1. The pressure oscillograms recorded for these discharges by means of a broadband hydrophone and reduced to dimensionless form are shown in Fig. 2 (curves 1 and 2, respectively). Curve 3 in the same figure shows the reduced pressure pulse for discharge 3. The dashed curve represents the theoretical pulse.

The theoretical curves for W_a, W and A for discharge 2 are shown in Fig. 3. This figure also shows the energy E released in the discharge channel. The dashed curve represents the sum $W_a + W + A$.

Figures 4 and 5 show the theoretical and experimental (from the data of [8] relationships between the amplitude of the pressure pulse and the capacitance $n = C/C_0$ of the storage device and the voltage on it for a distance of 90 cm from the discharge. The capacitance C of each capacitor in the bank was 7.5 μF.

For the discharge 1 we measured the first period T^* of pulsation of the gas bubble near the hard reflector at a depth of 0.5 m from the water surface. The value of $T°$ was 12.9 msec. The value of $T°$, calculated for these conditions on the assumption that the gas in the bubble after the discharge remains in the atomic state ($\gamma_1 = 1.66$), is 16.6 msec. Accordingly, the theoretical value of the bubble energy is 37%, and the experimental value 22%, of the energy released in the channel. The theoretical value of the electroacoustic efficiency is $\eta = 12\%$ and the experimental value is $\eta = 14\%$.

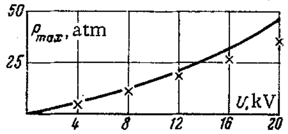

Fig. 5

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